An Optimization Study for the Bow Form of High Speed Displacement Catamarans

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Abstract

Interest in high-speed marine transportation has acquired a wide variety of hydrodynamic research activities on catamarans, including shape optimization for minimum total resistance. This study attempts to utilize the mathematical programming in optimizing the bow form of twin hulls for minimum total resistance as well as in analyzing the optimization by physical model tests. The total resistance is assumed to be composed of wave-making and frictional components, which are formulated using Michell’s thin-ship theory and ITTC-1957 friction line, respectively. The optimized hull form is analyzed by means of a computational flow solver before going through the experimental analysis. The study demonstrates the capabilities of the optimization procedure, presented for catamarans, in reducing the total resistance, as well as its limitations to be used as a design tool in a relatively high-speed zone.

1. INTRODUCTION

Recent upsurge in their commercial applications indicates that high-speed craft are gradually becoming ships of interest for ship owners, engineers and researchers as the importance of speed increases in the seaborne trade. It appears that catamarans, and probably trimarans in the near future, can satisfy the requirements of marine transportation especially those of passenger ferry market as the speed and safety are the main concerns. Therefore, a better understanding of relatively complex resistance characteristics of twin-hulls is of crucial importance on the one hand, developing an optimization procedure of these hull forms is an indispensable goal on the other.

An investigation into the components of resistance of catamarans has been addressed by Insel and Molland [1]. Meanwhile a number of computational investigations of the wave resistance of multihulls have been reported recently in the literature, such as [2], [3], [4]. A contribution to the problem of optimizing the forebody geometries of catamarans to minimize the resistance was first made by Hsiung and Xu[5], followed recently by Doctors and Renilson [6], who studied the influence of demihull separation and river banks and by Papanikolaou et al. [7], who handled the shape hull optimization problem as a component in a global computer aided optimization procedure. Hsiung and Xu[5] uses Lunde’s [8] formulation for the wave resistance and ITTC-1957 formula for the frictional resistance in formulating the objective function and then solve the convex quadratic programming problem with linear inequality constraints to obtain optimal forebody shapes for catamarans. In their study the Froude numbers chosen for optimization are not higher than 0.312 and the experimental data are given only to compare the effects of demihull separation. In the work of Papanikolaou et al. [7], the total optimization is treated by the method of Lagrange’s multipliers where the design constraints must be linear equalities. An extension of the Michell integral is also made in [7] by a normal dipole distribution on the centerplane to include asymmetric demihull forms into the wave resistance analysis of catamarans. Meanwhile the efforts to minimize the wave resistance of catamarans lead, on the other hand, to some unconventional solutions of cambered catamarans by Chen [9] and of staggered catamarans by Söding [10].

In the current work bow form optimization of high-speed displacement-catamarans is tackled by extending the previous work of Gören et al. [11]. The basic purpose of this study is to test and to ascertain the capabilities of the optimization procedure based on convex quadratic programming problem as a result of the use of thin ship theory, supported by a computational wave resistance analyzer, through a series of experimental analysis. The total resistance is decomposed into skin friction resistance, calculated by ITTC-1957 formula, and wave resistance based on Lunde’s [8] formulation. The expression of total

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resistance is then taken as the objective function, which is quadratic in ship’s half breadths. The linear inequality constraints used in the quadratic programming problem allow to implement the necessary design constraints into the procedure. The standard quadratic programming problem is then solved by Wolfe’s [12] algorithm. Although it is a well-known fact that in the high speed zone, the percentage of the wave pattern resistance in total resistance does not generally exceed 15%, the present study points out that by means of the optimization procedure it is possible to attain up to 5% gains in total resistance which may be regarded as considerable for catamarans operating at high Froude numbers greater than 0.50.

2. MATHEMATICAL PROGRAMMING PROBLEM FOR TWIN HULLS

The previous works of Gören and Calisal [13] and Gören et al. [11] is extended to cover twin hull geometries. Lunde’s [8] well-known formulation is used to express twin hull wave resistance based on thin-ship theory;

\[
R_w = 4 \rho c^2 \int_0^T \left( \frac{(u^2 + 1)^2}{u(u^2 + 2)} \right) \frac{2 + \cos \left( \frac{2h g}{c^2} (u^2 + 1)u(u^2 + 2) \right)}{\sqrt{u(u^2 + 2)}} du
\]

(1.a)

where \( c \) is the ship’s speed, \( \rho \) is the density of water and \( g \) is the gravitational acceleration. \( L \) is the length of the ship, \( T \) is the draft and \( F_z \) denotes the derivative of the hull surface with respect to \( x \). The coordinate axis used in the formulation is depicted in Figure 1.

Figure 1. The Coordinate axis and the arrangement of the demihulls

One can discretize the underwater geometry by tent functions (see Hsiung [14] for details). Thus (1.a) and (1.b) is then nondimensionalized and given in terms of the half breadths, \( y_i \), by the use of tent functions;

\[
C_w = \sum_{m=1}^{n} \sum_{n=1}^{m} y_{m,n} y_{n} \textbf{D} \textbf{y}
\]

(2)

where \( \textbf{D} \) denotes the total number of half-breadths since \( I \) is the number of waterlines while \( J \) is the number of stations. The coefficient matrix \( \textbf{D} \) is derived adopting the same approach in Hsiung [14]. The quantity calculated by equation (2) may not be considered as a good approximation of the wave resistance for high speed vessels with transom stern. However this value, which is of the same order as wave resistance, can relatively be minimized within the linear theory formulation.

The other component of the resistance that appears in the objective function is the equivalent flat-plate frictional resistance, which depends on the wetted surface area approximated as

\[
S = 2 \int_{S_0} \left[ 1 + \frac{1}{2} F_z^2(x,z) + \frac{1}{2} F^2_z(x,z) \right] dxdz
\]

(3)

Where, \( S_0 \) is the projection area of the wetted surface of the ship on the center-plane. \( F_z \) is the derivative with respect to \( z \). The use of tent functions is again made in (3) to give the frictional resistance coefficient, which is based on the ITTC-1957 formula, in a quadratic form in terms of the ship half breadths as in the following form:

\[
C_F = c_0 + c_0 y + y^T A_F y
\]

(4)

In accordance with the assumption made for the total resistance, the objective function used in the optimization procedure is taken as the sum of wave resistance (2) and the frictional resistance (4):

\[
C_T = C_F + C_W = c_0 + c_0 y + y^T C y
\]

(5)

Note that \( C \) is a positive semi-definite, that is a symmetric matrix.

The quadratic character of the objective function implies the employment of the quadratic programming in which the design constraints can be given by a set of inequalities. Thus the general form of the quadratic programming is expressed as

\[
\text{to minimize } \textbf{p} \textbf{y} + \textbf{y}^T \textbf{C} \textbf{y}
\]

(6)

to satisfy \( \textbf{A} \textbf{y} \leq \textbf{B} \) with \( y_i \geq 0 \)


3. NUMERICAL STUDY FOR OPTIMIZATION

Before going through the optimization process for twin hulls, equation (1.a) needs a special numerical treatment, since the integrand includes a highly oscillatory term. This is overcome by a novel numerical technique proposed by Sidi [15]. Any desired accuracy can be attained by increasing the number of recursive steps in Sidi’s algorithm. The intermediate integrations
between the roots of the cosine term are done by Gaussian quadratures.

Since the primary goal of the present study is to optimize the bow form of the demihulls, no effort has been spent to determine the optimum hull spacing, \( s=2b \). However, the hydrodynamic interaction between the two demihull is taken into account in the wave resistance formulation. As stated in the introduction section, this kind of an investigation can be found in [5] for moderate Froude numbers. Therefore 3 different “hull spacing to length” ratios, which provide a practical range of importance for catamarans in service, \( s/L=0.2, 0.3 \) and \( 0.4 \), are chosen. Optimization efforts have been focused on the spacing ratio, \( s/L=0.3 \).

An NPL based form, Bailey [16], is selected as the form to be optimized. The main characteristics of this NPL form are given in Table I.

| \( L_{\text{wl}} \) | 25.25 m |
| \( B_{\text{max}} \) | 2.6 m |
| \( T \) | 1.4 m |
| \( C_{B} \) | 0.410 |

The forebody volume of the original NPL form, which undergoes the optimization procedure, was chosen as the volume between bow profile and the \( 18^{\text{th}} \) station. The half-breadths on the stations 20 (F.P.), \( 19^{1/2} \), 19 and 18, each being described by 6 waterlines were the unknowns in the optimization study. The resistance coefficient matrices in (5) were computed only for the design waterline at \( T=1.4 \) m and at the optimization speed of 20 knots which corresponds \( F_{n}=0.65 \).

Among the set of design constraints, protruding length of the bulb and the forebody volume were employed as the variable design constraints, while the other constraints such as a limiting value for waterline slopes, waterplane area coefficient were kept constant. Two protruding lengths, namely, \( l_{P}=1.0 \) m and \( l_{P}=1.5 \) m, were taken into consideration while forebody block coefficient \( C_{Bf} \) was increased gradually from an acceptable minimum for every \( l_{P} \). As a consequence of the solution of the system (6) for every combination of the set of design constraints, a matrix of optimum hull forms were obtained. Eventually a corresponding matrix of numerical evaluation of the total resistance by means of equation (5) was calculated. Ultimately the optimized form having the best resistance performance was selected from the matrix. In Figure 2a and 2b the cross-sectional curves of the original and optimal hull forms (lines faired) can be seen. This form proposed for tank testing, has a protruding bulbous bow with a protruding length of \( l_{P}=1.5 \) m and the total volume was increased by 5% as compared to the original NPL form.

Before the model experiments were conducted, more accurate and reliable numerical tests and comparisons were done by a computational wave resistance flow solver reported in Goren and Atlar [17] based on Dawson’s [18] algorithm. A sample of geometric modeling of the optimized hull form as it was used in the flow solver is given in Figure 3. It is found from the computed results that 12% of gain is expected at least in the wave resistance at \( F_{n}=0.65 \) due to the optimal form, with \( s/L=0.3 \). The same optimized form shows 27% reduction in wave resistance at \( F_{n}=0.4 \).

The optimal demihull and the original NPL hull were also compared with respect to their wave deformations along the outer side of the demihull in Figure 4. In Figures 5.a and 5.b computed contour plots of the wave pattern of the original and optimal demihulls are shown, respectively. The improvement in the wave resistance is obvious due to relative reduction in the bow wave elevation of the optimal form, both quantitatively and qualitatively, as shown in Figures 4 and 5.
Figure 3. Geometric modeling of optimized demihull and its free-surface vicinity.

Figure 4. Computed wave elevations along the outer side of the demihull (Fn=0.65).

Figure 5a. Computed wave pattern around the original demihull (Fn=0.65).

Figure 5b. Computed wave pattern around the optimal demihull (Fn=0.65).
Figure 7. Comparison of experimental total resistance coefficients for original and optimized models.
Figure 8. Measured wave spectrums for original and optimized forms, \( F_n = 0.65 \).
4. EXPERIMENTAL WORK

In parallel to the numerical investigation, the physical model tests were carried out to investigate the resistance characteristics of the original NPL form and the optimized hull form in Ata Nutku Ship Model Testing Laboratory (160m x 6.0m x 3.40m) of I.T.U., which is an ITTC Class Facility. The experiments were performed for two sets of measurements, namely, total resistances and wave pattern resistances.

Associated models were built with a length of 2.5 meters and tripwires were used for the turbulence stimulation, as shown in Figure 6. The test cases covered were the mono hull (i.e. one demihull in isolation) and twin hulls having s/L ratios of 0.2, 0.3 and 0.4, [19].

![Figure 6. Bow form view of the two models: Original NPL form and its optimized counter part (at the back).](image)

The measurements in total resistance tests were acquired by using electronic dynamometer and by means of computer aided data acquisition system, for a range of Froude numbers of 0.1≤Fn≤0.7. Models were tested free to sink and to trim. The results of the total resistance tests are presented in Figure 7. As shown in this figure, up to 24% of reductions can be observed in total resistance, C_T, for the mono-hull configuration beside the fluctuations around the low speed range. The reduction in total resistance attains its maximum value around Fn=0.5. For the twin-hull configurations, reductions in wave resistance are observed for s/L=0.2 as 10%, s/L=0.3 as 11% and for s/L=0.4 as 12% at the speed which corresponds to Fn=0.65. Note that optimization was performed for Froude number 0.65 and for the gap ratio s/L=0.3. In order to make a direct comparison of wave resistance with that of the numerical analysis, wave patterns generated by the twin hulls were also measured by 4 wave probes located at one side of the tank so as to give the longitudinal cut data. The wave cut analysis depends on the wave form expression:

\[ \zeta = \sum_{n=0}^{N} \left[ \eta_n \cos(\omega_n x + \varphi_n) + \eta_n \sin(\omega_n x) \right] \cos \left( \frac{2\pi n}{W} \right) \]  \( (7) \)

where \( W \) is the width of the tank, \( \omega_n = K_n \cos(\theta_n) \),
\[ K_n \sin(\theta_n) = \frac{2\pi n}{W} \quad \text{and} \quad K_n - \frac{g}{\nu^2} \sec^2(\theta_n) = 0. \]

Here n, K_n, and \( \theta_n \) denote the n^{th} harmonic, wave number and wave angle, respectively. Thus, wave pattern resistance is given by:

\[ R_{WP} = \frac{\rho g W}{2 \pi} \left( \sum_{n=0}^{N} \left[ \eta_n \cos(\omega_n x + \varphi_n) + \eta_n \sin(\omega_n x) \right] \cos \left( \frac{2\pi n}{W} \right) \right) \]  \( (8) \)

In this case, resistance components are expressed as: \( R_T = R_W + R_{WP} = (1+k)R_T + R_{WP} \), \((1+k)\) is being the form factor. The results of the wave pattern resistance are shown in Figure 8 in terms of wave spectra only for the case of s/L=0.3. The optimal form reduced the wave pattern for the wave angles up to 60°. It is also effective for all tested demihull separations. In Figure 9, it is possible to compare the numerical and experimental results for the gap ratio s/L=0.3. It should be remarked here that there is 5% excess of wetted surface area in optimized form as compared to that of the original form. The experimental and numerical analyses give the same order of reductions in wave pattern resistance, where as a shift is observed at crest and troughs of the resistance curves. This may be attributed to the inherent phase shifts in Dawson’s algorithm, which appear considerably when there is interference. It is interesting to note here that the present optimization is more successful in reducing divergent waves rather than reducing the interference effects in high-speed zone. This can be seen in Figure 10. Ultimately, the effective power requirements of the original form and the optimized form for s/L=0.3 are given in Figure 11 for comparison. It is clear that there is at least 4% reduction in required effective power at Fn=0.65

5. CONCLUSION

An attempt is made to optimize the bow forms of high-speed displacement catamarans. The optimization procedure presented here is able to produce reasonable protruding bulbs suitable for catamarans. The numerical and experimental studies show that the optimized bow form is effective in reducing the wave resistance successfully. The results show that at least 4% of reduction is attainable in total resistance which may be regarded as considerable for catamarans operating at high Froude numbers greater than 0.50.

On the other hand experimental work points out that the resistance reductions are attained mostly due to the reductions in divergent wave patterns. Although it is known that asymmetric forms may be helpful in reducing interference resistance, a further study is required to search for obtaining reductions in the interference resistance as well.
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